Math 45 SSM 2/e 6.5 Summary of Factoring Techniques

Objective 1) Factor completely
- no new methods
- more than one method needed
- which method to do first?

step 0: Write in standard form. (If more than one variable, choose one.)

step 1: Factor out any monomial or polynomial GCF. (6.1)
If leading coefficient is negative, factor out GCF w/ -1.

step 2: Count the terms.

Remember
- Terms are separated by + or − ; ignore any + or − inside parentheses.
- An expression is completely factored when it is one term.

step 3:

If 4 terms: use grouping (6.1)
{ GCF 2 terms
{ GCF 2 terms
polynomial GCF

If 3 terms: • Is it a perfect square trinomial? (6.4)

$$a^2 - 2ab + b^2 = (a-b)^2$$
$$a^2 + 2ab + b^2 = (a+b)^2$$

• Is its leading coefficient 1? (single X) (6.2)

$x^2 + bx + c$ product c; q_1, q_2; sum b

$(x+q_1)(x+q_2)$

• Is its leading coefficient $\neq 1$? (double X or guess and check) (6.3)

$ax^2 + bx + c$ $a \cdot c$; q_1, q_2; b

$ax^2 + q_1x + q_2x + c$ then group.

If 2 terms: • Is it a difference of 2 squares? (6.4)

$$a^2 - b^2 = (a-b)(a+b)$$

• Is it a sum of 2 squares? (6.4)

$a^2 + b^2$ is prime

→

(2 terms cont)

- Is it a difference of two cubes? (6.4)
$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

- Is it a sum of two cubes? (6.4)
$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

step 4: Check:
- The expression should be one term.
- No factor should have a GCF.
- Any GCF taken out in step 1 should be written in the final answer
- No factor is a diff of squares.
- No factor can be factored
- final answer is "prime" only if no factoring of any type (including GCF) can be done at any step.

step 5: If time permits, check by multiplying.

Potential Complications.

1) The terms are out of order.
Choose a variable with different exponents in each term and write in descending order by the exponents on that variable.

2) The leading coefficient is negative.
Factor out a -1 with GCF.

3) It looks factorable, but can't find numbers that work.
List all potential factors to be sure.
Expression is prime.

4) It has more than one variable.
Does every term have both variables?
Factor out GCF.

① Example $10x^3y^4z + 15z^5y + 25x^2y^7z^3$
$= \boxed{5x^2y(2xy^3z + 3x^3 + 5y^6z^3)}$

cont →

Complications, cont.
(More than one variable, cont)

- Does only the middle term (of 3 terms) have both variables?

(2) Example: $x^2 + 4xy - 12y^2$

only middle term has both x and y.

Write variable from 1st term at the front of each factor

$(x \quad)(x \quad)$

and variable from last term at the back of each factor

$(x \quad y)(x \quad y)$

Find numbers using $\times$ or $\times\!\!\times$ (depending on leading coef).

or guess and check.

-12, 6, -2, 4

$$(x + 6y)(x - 2y)$$

- Do both variables appear in two terms but not the third term?

(3) Example: $6x^2y^2 - 13xy + 6$

Write variables from 1st term at the front of each factor.

$(xy \quad)(xy \quad)$

Find numbers using $\times$ or $\times\!\!\times$ (depending on leading coef).

or guess and check.

36, -9, -4, -13

$6x^2y^2 - 9xy - 4xy + 6$ ←

CAUTION: Use both variables in both rewritten terms.
$-9xy$ and $-4xy$

$3xy(2xy - 3) - 2(2xy - 3)$

$$(2xy - 3)(3xy - 2)$$

Complications con't

5) After factoring, factors can be factored.

④ Example $4x^3 - 6x^2 - 36x + 54$

GCF $= 2(2x^3 - 3x^2 - 18x + 27)$

grouping $= 2[x^2(2x-3) - 9(2x-3)]$

$= 2(2x-3)(x^2-9)$

diff of sq $= \boxed{2(2x-3)(x-3)(x+3)}$

⑤ Example: $80x - 5x^5$

standard form $-5x^5 + 80x$

monomial GCF with negative $-5x(x^4 - 16)$

diff of sq. $-5x(x^2-4)(x^2+4)$

diff of sq. $\boxed{-5x(x-2)(x+2)(x^2+4)}$

(Sum of sq x^2+4 prime)

Factor Completely.

⑥ $2n^3 - 10n^2 - 6n + 30$

GCF $= 2(n^3 - 5n^2 - 3n + 15)$

grouping $= 2[n^2(n-5) - 3(n-5)]$

$= \boxed{2(n-5)(n^2-3)}$

n^2-3 is almost a diff of sq., except 3 is not a square!
So (n^2-3) is prime.

⑦ $12x^2 + 36xy + 27y^2$

GCF $= 3(4x^2 + 12xy + 9y^2)$

perf. sq. $= \boxed{3(2x+3y)^2}$

check $2(2x)(3y) = 12xy$ ✓

(8) $24p^3q + 81q^4$

GCF $= 3q(8p^3 + 27q^3)$

Sum cubes $= 3q(2p+3q)((2p)^2 - (2p)(3q) + (3q)^2)$

$= \boxed{3q(2p+3q)(4p^2 - 6pq + 9q^2)}$

Note: If done correctly, the trinomial part of a sum (or difference) of cubes is prime. ($4p^2 - 6pq + 9q^2$ is prime.)

(9) $29a^2 + 5a^4 + 20$

standard form: $= 5a^4 + 29a^2 + 20$

$= 5a^4 + 25a^2 + 4a^2 + 20$

$= 5a^2(a^2+5) + 4(a^2+5)$

$= \boxed{(a^2+5)(5a^2+4)}$

5·20
100
25 4
29

✕ : leading coef $5 \neq 1$

(10) $-35x + 45 - 40x^2$

standard form: $= -40x^2 - 35x + 45$

GCF $= \boxed{-5(8x^2 + 7x - 9)}$

✕

no pair that multiplies to 72 will add to 7.
$8x^2 + 7x - 9$ is prime.

−72
7

−1, 72
−2, 36
−3, 24
−4, 18
−6, 12
−8, 9

Note: $-35x + 45 - 40x^2$ is not prime, because we were able to factor out a GCF (−5).

(11) $49 - x^2$

MathXL will accept $\boxed{(7-x)(7+x)}$ and I will, too.

But for the purposes of chapter 7, let's do this in standard form:

$= -x^2 + 49$

GCF $= -(x^2 - 49)$

$= \boxed{-(x-7)(x+7)}$

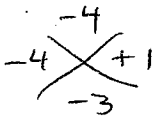
Note: The negative is essential: $(x-7)(x+7)$ is NOT the same as $(7-x)(7+x)$. (Multiply by FOIL to check.)

(12) $4n^2 - n^4 + 3n^3$

standard form: $= -n^4 + 3n^3 + 4n^2$

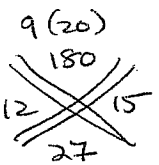
GCF w/ -1 : $= -n^2(n^2 - 3n - 4)$

X single x : $= \boxed{-n^2(n-4)(n+1)}$



(13) $18x^2y^2 + 54xy^2 + 40y^2$

GCF $= 2y^2(9x^2 + 27x + 20)$



1, 180
2, 90
3, 60
4, 45
5, 36
6, 30
9, 20
10, 18
12, 15

X or guess + check $= 2y^2(9x^2 + 12x + 15x + 20)$

$= 2y^2[3x(3x+4) + 5(3x+4)]$

$= \boxed{2y^2(3x+4)(3x+5)}$

(14) $9x^2 - 1$

diff of sq: $\boxed{(3x-1)(3x+1)}$

(15) $9x^2 - 9x$

GCF $= \boxed{9x(x-1)}$